(54) Title: PORTFOLIO OPTIMIZATION BY MEANS OF RESAMPLED EFFICIENT FRONTIERS
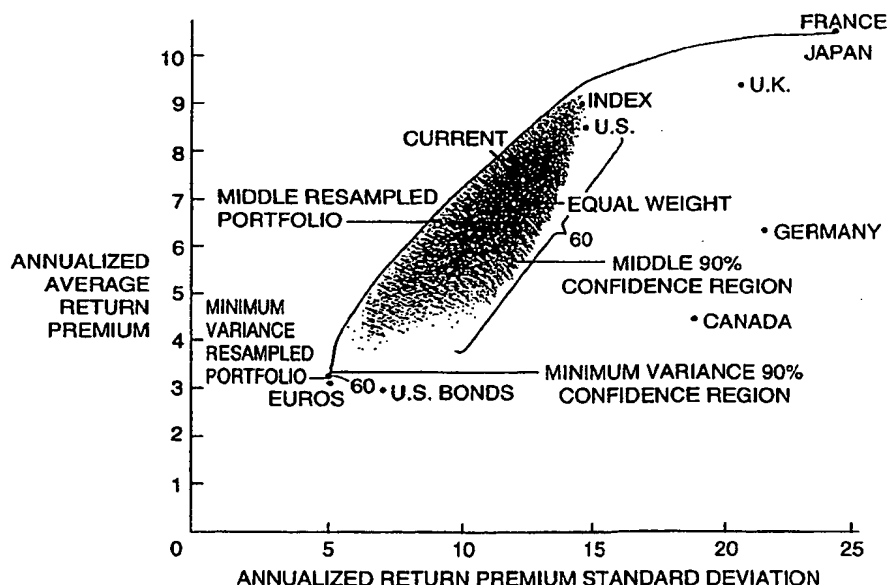
(57) Abstract

A method for evaluating an existing or putative portfolio having a plurality of assets. A mean–variance efficient portfolio is computed for a plurality of simulations of input data statistically consistent with an expected return and expected standard deviation of return, and each such portfolio is associated, by means of an index, with a specified portfolio on the mean variance efficient frontier. A statistical mean of the index–associated mean–variance efficient portfolios is used for evaluating a portfolio for consistency with a specified risk objective.

Portfolio Optimization by Means of Resampled Efficient Frontiers

Technical Field

The present invention relates to a method for selecting a portfolio of tangible or
5   intangible assets subject to optimization criteria yielding an enhanced mean-variance
efficiency.

Background of the Invention

Managers of assets, such as portfolios of stocks, projects in a firm, or other
10   assets, typically seek to maximize the expected or average return on an overall
investment of funds for a given level of risk as defined in terms of variance of return,
either historically or as adjusted using techniques known to persons skilled in portfolio
management. Alternatively, investment goals may be directed toward residual return
with respect to a benchmark as a function of residual return variance. Consequently, the
15   terms "return" and "variance," as used in this description and in any appended claims,
may encompass, equally, the residual components as understood in the art. The capital
asset pricing model of Sharpe and Lintner and the arbitrage pricing theory of Ross are
examples of asset evaluation theories used in computing residual returns in the field of
equity pricing. Alternatively, the goal of a portfolio management strategy may be cast as
20   the minimization of risk for a given level of expected return.

The risk assigned to a portfolio is typically expressed in terms of its variance σ_P^2
stated in terms of the weighted variances of the individual assets, as:

$$\sigma_P^2 = \sum_i \sum_j w_i w_j \sigma_{ij} \quad ,$$

25

where w_i is the relative weight of the i-th asset within the portfolio,

$$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$$

30   is the covariance of the i-th and j-th assets, ρ_{ij} is their correlation, and σ_i is the standard
deviation of the i-th asset. The portfolio standard deviation is the square root of the
variance of the portfolio.

Following the classical paradigm due to Markowitz, a portfolio may be optimized, with the goal of deriving the peak average return for a given level of risk and any specified set of constraints, in order to derive a so-called "mean-variance (MV) efficient" portfolio using known techniques of linear or quadratic programming as

5   appropriate. Techniques for incorporating multiperiod investment horizons are also known in the art. As shown in FIG. 1A, the expected return μ for a portfolio may be plotted versus the portfolio standard deviation σ, with the locus of MV efficient portfolios as a function of portfolio standard deviation referred to as the "MV efficient frontier," and designated by the numeral 10. Mathematical algorithms for deriving the

10   MV efficient frontier are known in the art.

Referring to FIG. 1B, a variation of classical Markowitz MV efficiency often used is benchmark optimization. In this case, the expected residual return α relative to a specified benchmark is considered as a function of residual return variance ω, defined as was the portfolio standard deviation σ but with respect to a residual risk. An investor

15   with portfolio A desires to optimize expected residual return at the same level ω_A of residual risk. As before, an efficient frontier 10 is defined as the locus of all portfolios having a maximum expected residual return α of each of all possible levels of portfolio residual risk.

Known deficiencies of MV optimization as a practical tool for investment

20   management include the instability and ambiguity of solutions. It is known that MV optimization may give rise to solutions which are both unstable with respect to small changes (within the uncertainties of the input parameters) and often non-intuitive and thus of little investment sense or value for investment purposes and with poor out-of-sample average performance. These deficiencies are known to arise due to the propensity

25   of MV optimization as "estimation-error maximizers," as discussed in R. Michaud, "The Markowitz Optimization Enigma: Is Optimized Optimal?" Financial Analysts Journal (1989), which is herein incorporated by reference. In particular, MV optimization tends to overweight those assets having large statistical estimation errors associated with large estimated returns, small variances, and negative correlations, often resulting in poor ex-

30   post performance.

2

Summary of the Invention

In accordance with one aspect of the invention, in one of its embodiments, there
is provided a method for evaluating an existing or putative portfolio having a plurality of
assets. The existing portfolio is of the kind having a total portfolio value, where each

5   asset has a value forming a fraction of the total portfolio value, each asset has a defined
expected return and a defined standard deviation of return, and each asset has a
covariance with respect to each of every other asset of the plurality of assets. The method
has the steps of:

       a.   computing a mean-variance efficient frontier based at least on input data

10   characterizing the defined expected return and the defined standard deviation of return of
each of the plurality of assets;

       b.   indexing a set of portfolios located on the mean variance efficient frontier;

       c.   resampling a plurality of simulations of input data statistically consistent with
the defined expected return and the defined standard deviation of return of each of the

15   plurality of assets;

       d.   computing a simulated mean-variance efficient portfolio for each of the
plurality of simulations of input data;

       e.   associating each simulated mean-variance efficient portfolio with a specified
portfolio of the set of indexed portfolios for creating a set of identical-index-associated

20   mean-variance efficient portfolios;

       f.   establishing a statistical mean for each set of identical-index-associated mean-
variance efficient portfolios, the plurality of statistical means defining the resampled
efficient frontier; and

       g.   comparing the existing portfolio with a portfolio from the resampled efficient

25   frontier characterized by a specified risk objective.

In accordance with an alternate embodiment of the present invention, an index-
associated resampled fuzzy region may be identified in the risk/return plane
corresponding to portfolios having a norm, with respect to a corresponding portfolio on

30   the resampled efficient frontier, that is less than or equal to a value corresponding to a
specified confidence level. A current portfolio may be assessed with respect to required

3

optimization based at least on the value of the norm of the current portfolio relative to the index-associated resampled fuzzy region.

Brief Description of the Drawings

5   The invention will be more readily understood by reference to the following description, taken with the accompanying drawings, in which:

FIGS. 1A depicts the prior art principle of calculating an efficient frontier of maximum expected return for each given level of portfolio risk;

FIGS. 1B depicts the prior art principle of calculating an efficient frontier of

10   maximum expected residual return for each given level of portfolio residual risk;

FIG. 2 displays a set of statistically equivalent portfolios within the risk/return plane;

FIG. 3 displays statistically equivalent portfolios within the risk/return plane corresponding to three particular risk rankings on the efficient frontier: minimum

15   variance, maximum return, and a middle return portfolio;

FIG. 4 displays a region within the risk/return plane corresponding to statistically equivalent portfolios for a given level of probability;

FIG. 5 shows the resampled efficient frontier plotted in the risk/return plane in accordance with a preferred embodiment of the present invention; and

20   FIG. 6 displays a fuzzy region of statistically equivalent portfolios in the risk/return plane relative to resampled efficiency for two particular risk rankings for a given level of probability: minimum variance and middle return.

Detailed Description of Preferred Embodiments

25   The foregoing features of the invention will be more readily understood by reference to the following detailed description taken with the accompanying drawings.

In accordance with various embodiments of the present invention, it is recognized that MV optimization is a statistical procedure, based on estimated returns subject to a statistical variance, and that, consequently, the MV efficient frontier, as

30   defined above, is itself characterized by a variance. For a given set of input data characterizing the universe of assets to be considered to comprise a portfolio, and for a given set of constraints on the makeup of the portfolio, an MV efficient frontier may be

calculated, using standard techniques discussed above. Since the input data are of a statistical nature (i.e., characterized by means with associated variances and other statistical measures), the input data may be resampled, by simulation of optimization input parameters in a manner statistically consistent with the first set of data, as

5   described, for example, by J. Jobson and B. Korkie, "Estimation for Markowitz Efficient Portfolios," Journal of Portfolio Mangement, (1981), which is herein incorporated by reference. Other resampling methods are within the scope of the invention as described herein, and as claimed in any appended claims.

Based on the resampled data, the MV efficient portfolios may be recalculated,

10   subject to the same constraints as applied in the initial solution. In this manner, as shown in Fig. 2, a set 12 of statistically equivalent MV efficient portfolios may be calculated. By iterating this procedure, a large MV efficient "statistical equivalence" set of portfolios, in the expected return - portfolio variance space, may be generated. In particular, an existing portfolio may be found to be consistent, in a statistical sense, with

15   MV efficiency and thus not to require optimization, thereby potentially and advantageously saving transaction costs associated with revision of a portfolio. Referring to Fig. 3, statistically equivalent portfolios within the risk/return plane are shown corresponding to three particular risk rankings on the efficient frontier: namely, minimum variance 14, maximum return 16, and a middle return portfolios 18.

20   One method of establishing statistical equivalence is described in reference to FIG. 4. In accordance with one sample acceptance procedure, a sample acceptance area 20 is defined under efficient frontier 10. The $100(1-\alpha)\%$ sample acceptance region 20 includes, on average, the fraction $1-\alpha$ of Monte-Carlo estimated resampled portfolios, where α is the probability of a Type I error, i.e., the probability of rejecting the null

25   hypothesis when it is true, which are statistically equivalent to a portfolio denoted as CURRENT. The loci corresponding, in the risk/return plane, to particular assets included within the universe from which the sample portfolio is drawn are indicated in the figure. Thus, for $\alpha=0.1$, the 90% sample acceptance region 20 is obtained. Sample acceptance region 20 may be obtained, in practice, by a procedure in which the area under efficient

30   frontier 10 is divided into mutually exclusive column (or row) rectangles that include all the simulated portfolios. The base of the rectangle is defined as the minimum return point that contains the fraction $1-\alpha$ of the simulated portfolios within the rectangle. The

midpoints of the rectangles are connected to obtain curve 22 which constitutes the lower

boundary of sample acceptance area 20. Irregularities in the shape of curve 22 are

attributable in part to the small number of assets included in the example.

A disadvantage of the foregoing method is that all portfolios which are

5   statistically equivalent (in terms of the specified criterion) appear within sample

acceptance region 20 without accounting for desired risk objectives or other constraints.

Referring now to Fig. 3, a method is described whereby, in accordance with a preferred

embodiment of the present invention, all resampled portfolios within the risk/return

plane may be associated, many-to-one, with particular portfolios on MV efficient frontier

10   10. Various criteria may be applied in associating portfolios with those on the MV

efficient frontier, and all such associations are within the scope of the present invention.

As one example, each efficient frontier portfolio (i.e., each point on efficient frontier 10)

may be identified by its relative return rank. Similarly, the efficient frontier portfolios

may be ranked by their variance, the maximum variance corresponding to the maximum

15   return, the rankings by risk or return similarly mapping onto one another uniquely. Thus,

for example, the minimum variance portfolio 14 might have the lowest rank relative to

the other efficient portfolios of efficiency frontier 10. Similarly, maximum average

return portfolio 16 has the highest average return rank in each simulated efficient

frontier. Similarly, any other simulated portfolio is rank associated with a particular

20   efficient frontier portfolio. The sparsely clustered portfolios 18 shown in the figure

correspond to the 'middle' ranked efficient portfolio. In practice, the shape of the rank-

associated regions varies in dependence upon the position of the portfolio on the MV

efficient frontier.

It is not necessary, however, that the association with efficient frontier portfolios

25   be by rank, and particular portfolios on the MV efficient frontier may be indexed, and

thus index-associated, each with a set of statistically equivalent efficient portfolios lying

below the efficient frontier.

In accordance with an alternate embodiment of the invention, indexing of the set

of MV efficient portfolios for associating the resampled portfolios is to associate with

30   each MV efficient portfolio a "lambda value," defining the risk/return preference, with

respect to which the quantity $\phi = \sigma^2 - \lambda\mu$ is minimized, where σ^2 is the variance of each

portfolio and μ is the expected return of each portfolio of the set of portfolios located on

the mean variance efficient frontier. The parameter λ assumes a value between zero and infinity.

Once the ensemble of index-associated MV efficient portfolios has been associated, usual statistical measures of the ensemble may be derived. These measures

5   include, without limitation, the averages, standard errors, and t-statistics of the average of the portfolio weights of the rank-associated simulated efficient portfolios. Referring now to Fig. 5, an average of index-associated MV efficient portfolios may be defined, in accordance with preferred embodiments of the present invention, the average of index-associated MV efficient portfolios being referred to as a "resampled-efficient portfolio."

10   The average may be determined with respect to any of a variety of parameters, and, in accordance with a preferred embodiment, it is with respect to the vector average of the associated portfolios. The locus **40** of resampled-efficient portfolios is referred to as the "resampled efficient frontier." The resampled-efficient portfolio and its associated statistics may be applied as a statistical measure for portfolio analysis, as further

15   described below. Its application, as a choice for portfolio selection, advantageously removes, by definition, the "outlier" portfolios which strongly depend on values of a particular set of inputs and improves out-of-sample performance, on average. Statistical procedures for portfolio analysis and revision, and performance benefits based on these concepts, in accordance with embodiments of the present invention, are also described in

20   detail in the book, R. Michaud, *Efficient Asset Management*, (Harvard Business School Press, 1998), which is hereby incorporated by reference, as are all references cited in the text of the book.

In accordance with other embodiments of the invention, the portfolio optimization problem may be cast in terms of fuzzy sets and the application of fuzzy

25   logic. Referring now to Fig. 6, the "similarity" of a particular resampled efficient frontier portfolio (whether risk-indexed or otherwise indexed) to a corresponding resampled efficient portfolio may be defined by a "norm," with the norm having the usual properties of a distance function as known to persons skilled in the mathematical arts. Risk-indexing will be assumed in the following description solely for purposes of

30   coherence of presentation. In a preferred embodiment, the norm may be a Euclidean vector distance, or L_2 norm. The distribution of the values of the norms of risk-index associated resampled portfolios relative to the associated resampled efficient frontier

portfolio defines the distance criterion and fuzzy or similarity resampled efficient region (again, relative to the associated resampled efficient frontier portfolio) at a given level of probability. A fuzzy region, designated generally by numeral 60, corresponds to the region in the risk/return plane at probability level 0.9 using the Euclidean norm for the

5   minimum variance and middle resampled efficient risk-indexed portfolios, each taken relative to the resampled efficient frontier. In particular, the distance criterion for any portfolio P is taken to be the relative variance for portfolio P,

$$(P-P_0)^t \; * \; S \; * \; (P-P_0) \; ,$$

where $P-P_0$ is the difference vector of portfolio weights with respect to P_0, the corresponding index-associated portfolio on the resampled efficient frontier, and, S is the

10   input return covariance matrix (with the superscript 't' denoting the transpose of the difference vector). The norm is taken in the space of portfolio vectors (i.e., "portfolio space"). The distance criterion defined for probability level $1-\alpha$ is the value of the norm of the risk-indexed resampled portfolios corresponding to the $100(1-\alpha)$ percentile value of all risk-indexed resampled portfolios sorted from small to large. An example of the

15   use of a confidence region resampling test statistic relative to the resampled efficient frontier is provided at page 73 of Michaud, *Efficient Asset Management*. Portfolios within a risk-indexed resampled fuzzy region at a given level of probability may be said to be sufficiently similar to the risk-indexed resampled efficient portfolio to be considered resampled efficient and not to require optimization. Particular probability

20   levels and L_p norms may be specified as a matter of design choice, within the scope of the present invention.

In an alternative embodiment, the disclosed method for evaluating an existing or putative portfolio may be implemented as a computer program product for use with a computer system. Such implementation may include a series of computer instructions

25   fixed either on a tangible medium, such as a computer readable medium (*e.g.*, a diskette, CD-ROM, ROM, or fixed disk) or transmittable to a computer system, via a modem or other interface device, such as a communications adapter connected to a network over a medium. The medium may be either a tangible medium (*e.g.*, optical or analog communications lines) or a medium implemented with wireless techniques (*e.g.*,

30   microwave, infrared or other transmission techniques). The series of computer

8

instructions embodies all or part of the functionality previously described herein with respect to the system. Those skilled in the art should appreciate that such computer instructions can be written in a number of programming languages for use with many computer architectures or operating systems. Furthermore, such instructions may be

5   stored in any memory device, such as semiconductor, magnetic, optical or other memory devices, and may be transmitted using any communications technology, such as optical, infrared, microwave, or other transmission technologies. It is expected that such a computer program product may be distributed as a removable medium with accompanying printed or electronic documentation (*e.g.*, shrink wrapped software),

10   preloaded with a computer system (*e.g.*, on system ROM or fixed disk), or distributed from a server or electronic bulletin board over the network (*e.g.*, the Internet or World Wide Web). Of course, some embodiments of the invention may be implemented as a combination of both software (*e.g.*, a computer program product) and hardware. Still other embodiments of the invention are implemented as entirely hardware, or entirely

15   software (*e.g.*, a computer program product).

The described embodiments of the invention are intended to be merely exemplary and numerous variations and modifications will be apparent to those skilled in the art. All such variations and modifications are intended to be within the scope of the present invention as defined in the appended claims.

WE CLAIM:

1.   A method for selecting a value of portfolio weight for each of a specified plurality of assets of an optimal portfolio, the value of portfolio weight chosen from values between zero and unity, each asset having a defined expected return and a defined
5   standard deviation of return, each asset having a covariance with respect to each of every other asset of the plurality of assets, the method comprising:

    a.   computing a mean-variance efficient frontier based at least on input data characterizing the defined expected return and the defined standard deviation of return of each of the plurality of assets;

10   b.   indexing a set of portfolios located on the mean variance efficient frontier;

    c.   resampling a plurality of simulations of input data statistically consistent with the defined expected return and the defined standard deviation of return of each of the plurality of assets;

    d.   computing a mean-variance efficient portfolio for each of the plurality of
15   simulations of input data;

    e.   associating each simulated mean-variance efficient portfolio with a specified portfolio of the set of indexed portfolios for creating a set of identical-index-associated mean-variance efficient portfolios;

    f.   establishing a statistical mean for each set of identical-index-associated mean-
20   variance efficient portfolios, the plurality of statistical means defining the resampled efficient frontier; and

    g.   selecting a portfolio weight for each asset from the resampled efficient frontier according to a specified risk objective.

2.   A method according to claim 1, wherein the step of indexing the set of portfolios
25   includes associating a rank with each portfolio of the set of portfolios located on the mean variance efficient frontier.

3.   A method according to claim 1, wherein the step of indexing the set of portfolios includes associating a lambda value for which $\phi=\sigma^2-\lambda\mu$ is minimized, where σ^2 is the variance of each portfolio and μ is the expected return of each portfolio of the set of
30   portfolios located on the mean variance efficient frontier.

4.   A method for evaluating an existing portfolio having a plurality of assets, the existing portfolio having a total portfolio value, each asset having a value forming a

10

fraction of the total portfolio value, each asset having a defined expected return and a
defined standard deviation of return, each asset having a covariance with respect to each
of every other asset of the plurality of assets, the method comprising:

      a. computing a mean-variance efficient frontier based at least on input data
5   characterizing the defined expected return and the defined standard deviation of return of
each of the plurality of assets;

      b. associating a rank with each portfolio of assets associated with the mean
variance efficient frontier in such a manner as to create a plurality of ranked portfolios;

      c. resampling a plurality of simulations of input data statistically consistent with
10 the defined expected return and the defined standard deviation of return of each of the
plurality of assets;

      d.   computing a mean-variance efficient portfolio for each of the plurality of
simulations of input data;

      e. associating each simulated mean-variance efficient portfolio with a specified
15 portfolio of the set of indexed portfolios for creating a set of identical-index-associated
mean-variance efficient portfolios;

      f. establishing a statistical mean for each set of identical-index-associated mean-
variance efficient portfolios, the plurality of statistical means defining the resampled
efficient frontier; and

20   g. comparing the existing portfolio with a portfolio from the resampled efficient
frontier characterized by a specified risk objective.

5.   A method for evaluating an existing portfolio having a plurality of assets, the
existing portfolio having a total portfolio value, each asset having a value forming a
fraction of the total portfolio value, each asset having a defined expected return and a
25 defined standard deviation of return, each asset having a covariance with respect to each
of every other asset of the plurality of assets, the method comprising:

      a. computing a mean-variance efficient frontier based at least on input data
characterizing the defined expected return and the defined standard deviation of return of
each of the plurality of assets;

30   b. associating a rank with each portfolio of assets associated with the mean
variance efficient frontier in such a manner as to create a plurality of ranked portfolios;

11

c. resampling a plurality of simulations of input data statistically consistent with the defined expected return and the defined standard deviation of return of each of the plurality of assets;

d. computing a mean-variance efficient portfolio for each of the plurality of

5   simulations of input data;

e. associating each simulated mean-variance efficient portfolio with a specified portfolio of the set of indexed portfolios for creating a set of identical-index-associated mean-variance efficient portfolios;

f. establishing a statistical mean for each set of identical-index-associated mean-

10   variance efficient portfolios, the plurality of statistical means defining the resampled efficient frontier; and

g. associating a norm having a value characterizing a similarity of each portfolio with respect to a corresponding portfolio on the resampled efficient frontier;

h. defining a index-associated resampled fuzzy region corresponding to

15   portfolios having a norm value less than or equal to a value associated with a specified confidence level; and

i. evaluating whether a current portfolio requires optimization based at least on a norm of the current portfolio in comparison with the index-associated resampled fuzzy region.

20   **6.**   A computer program product for use on a computer system for selecting a value of portfolio weight for each of a specified plurality of assets of an optimal portfolio, the value of portfolio weight chosen from values between zero and unity, each asset having a defined expected return and a defined standard deviation of return, each asset having a covariance with respect to each of every other asset of the plurality of assets, the

25   computer program product comprising a computer usable medium having computer readable program code thereon, the computer readable program code including:

a. program code for computing a mean-variance efficient frontier based at least on input data characterizing the defined expected return and the defined standard deviation of return of each of the plurality of assets;

30   b. a sequencer for indexing a set of portfolios located on the mean variance efficient frontier;

12

c.   a routine for resampling a plurality of simulations of input data statistically
consistent with the defined expected return and the defined standard deviation of return
of each of the plurality of assets;

d.   program code for computing a mean-variance efficient portfolio for each of
5   the plurality of simulations of input data;

e.   program code for associating each simulated mean-variance efficient portfolio
with a specified portfolio of the set of indexed portfolios for creating a set of identical-
index-associated mean-variance efficient portfolios;

f.   a module for establishing a statistical mean for each set of identical-index-
10   associated mean-variance efficient portfolios, the plurality of statistical means defining
the resampled efficient frontier; and

g.   program code for selecting a portfolio weight for each asset from the
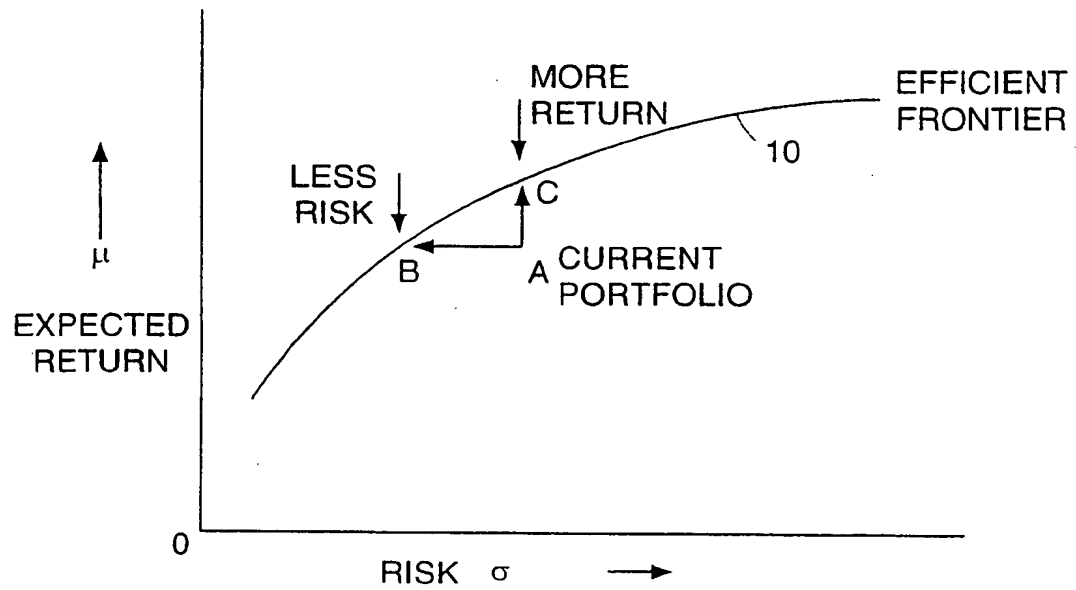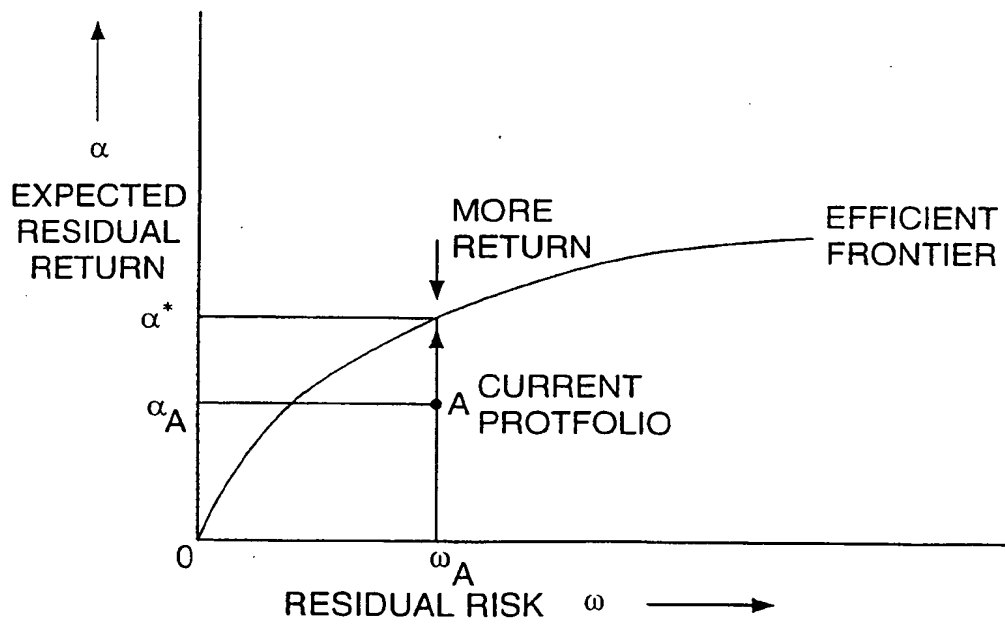resampled efficient frontier according to a specified risk objective.

13

1/6

MORE
RETURN

LESS
RISK

C

B   A CURRENT
PORTFOLIO

EFFICIENT
FRONTIER

10

μ

EXPECTED
RETURN

0

RISK σ

FIG. 1A
PRIOR ART

MORE
RETURN

EFFICIENT
FRONTIER

α

EXPECTED
RESIDUAL
RETURN

α^*

α_A

A CURRENT
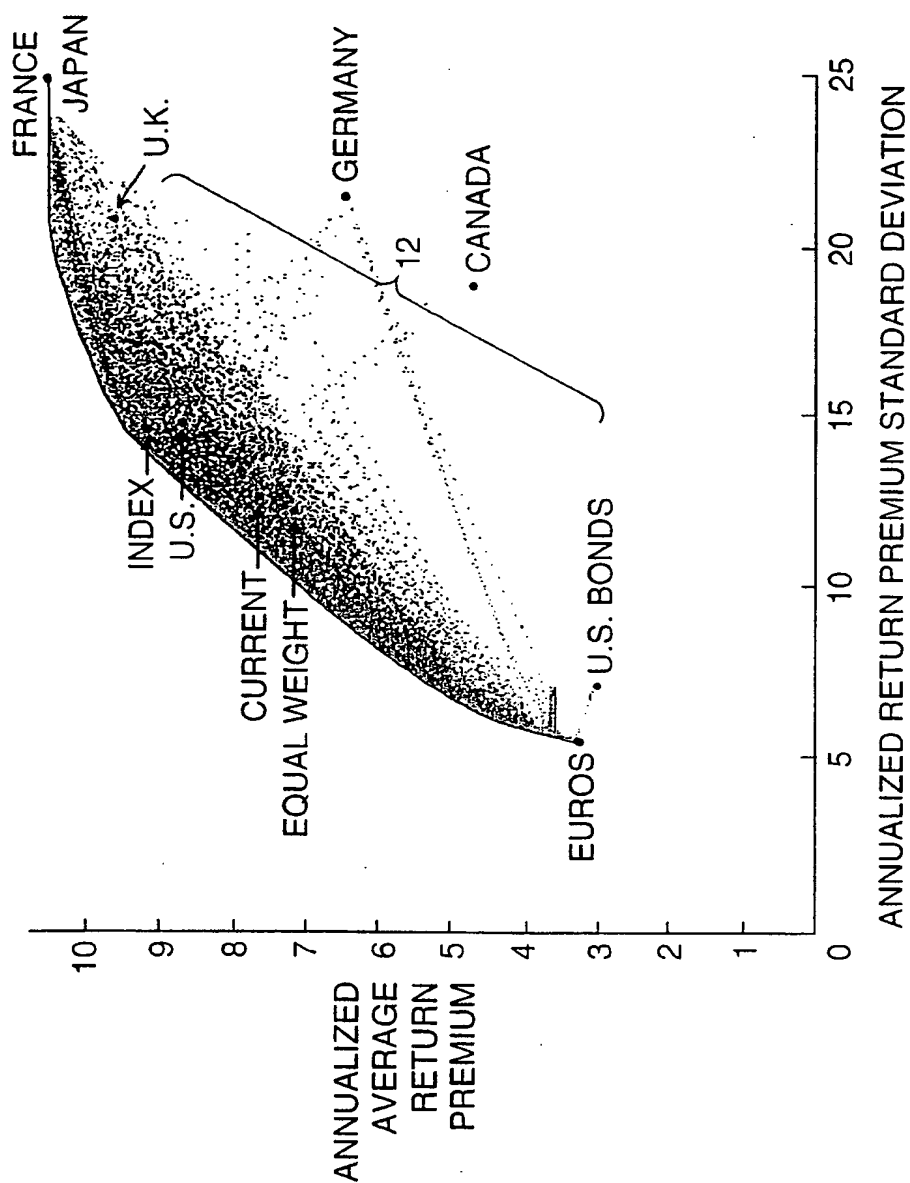PROTFOLIO

0   ω_A

RESIDUAL RISK ω

FIG. 1B
PRIOR ART

FIG. 2

FIG. 3

4/6
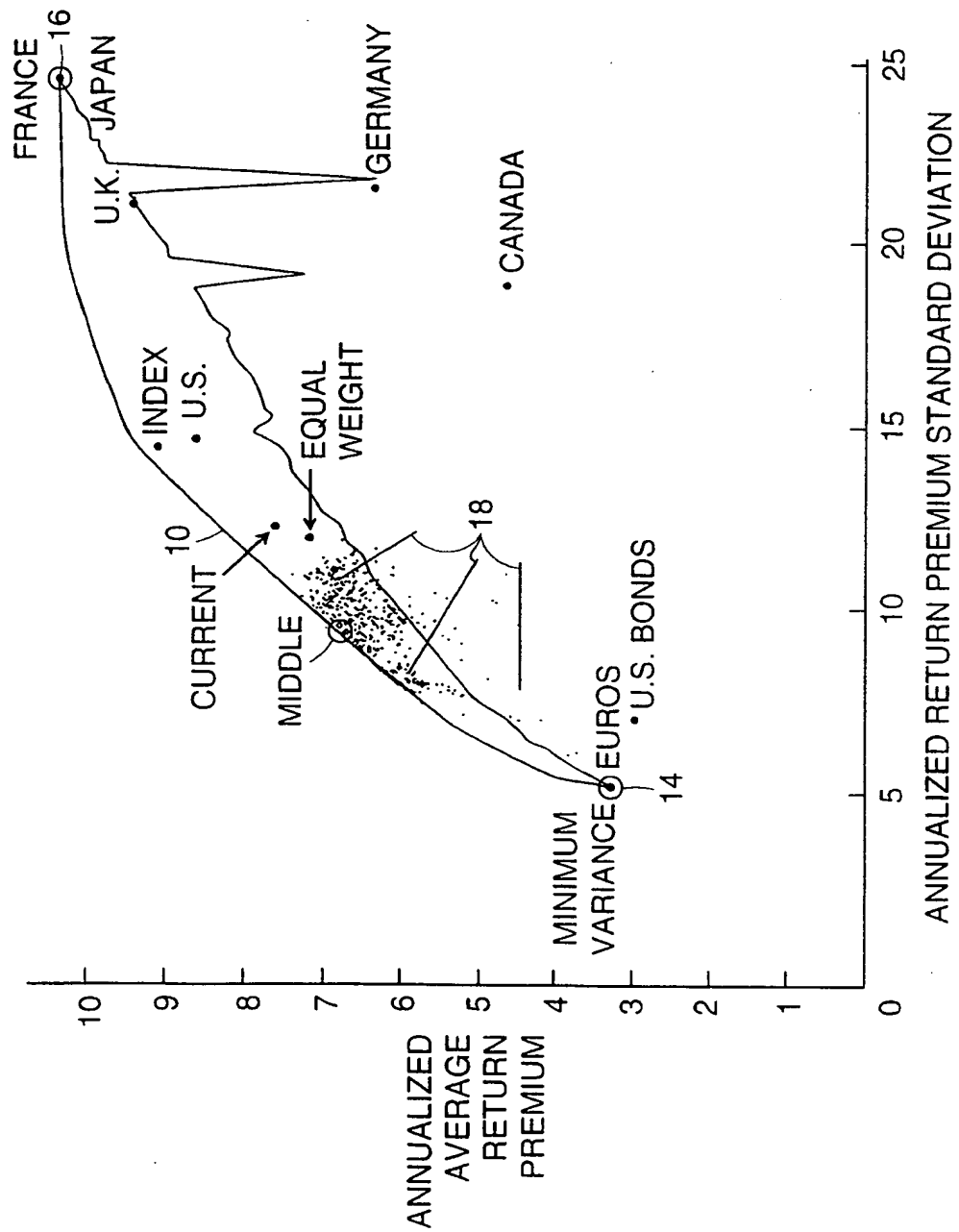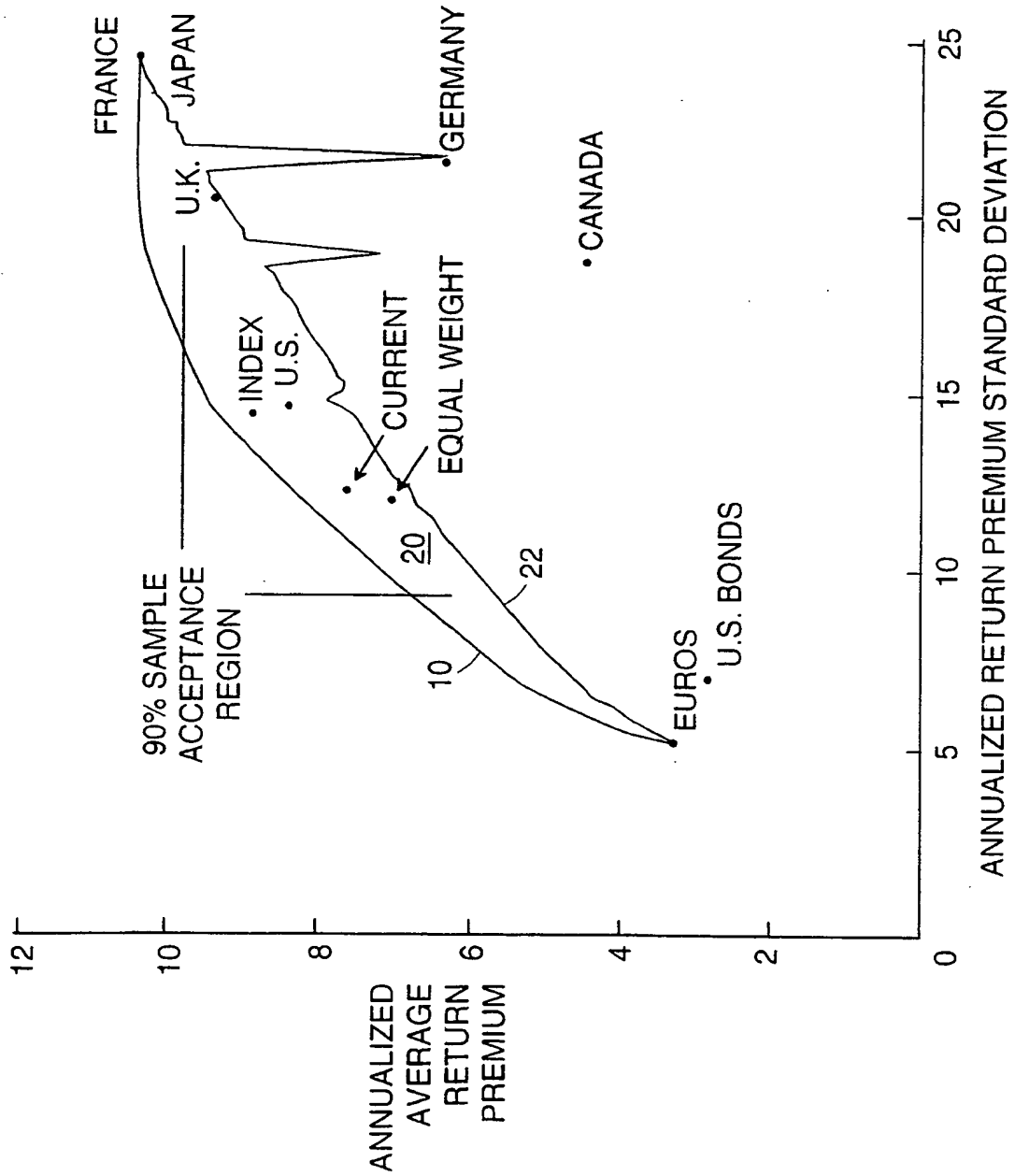
FIG. 4

5/6
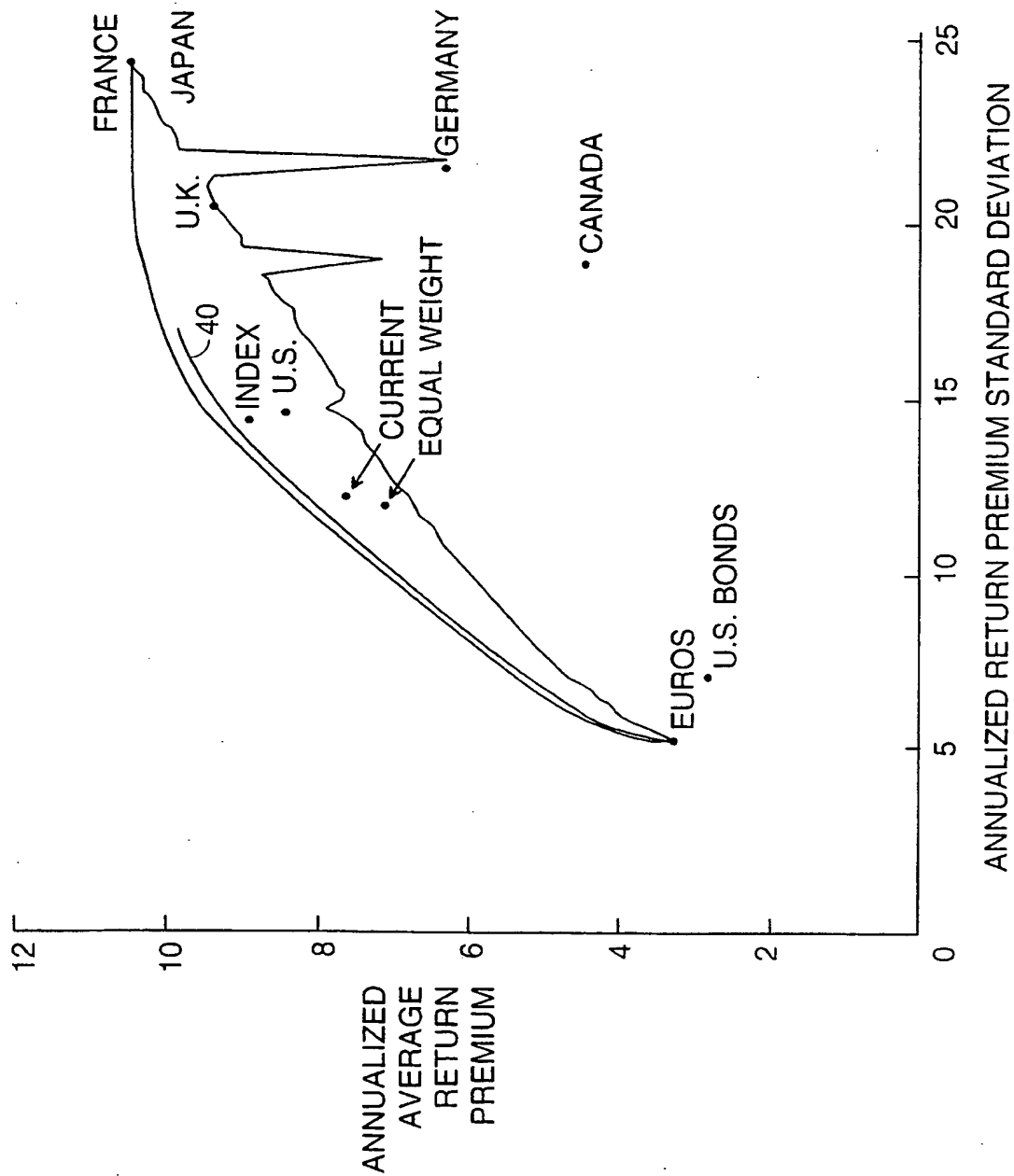
FIG. 5

6/6

FIG. 6

INTERNATIONAL SEARCH REPORT

A. CLASSIFICATION OF SUBJECT MATTER
IPC 6   G06F17/60

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

IPC 6   G06F

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category°	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	US 5 148 365 A (DEMBO RON S) 15 September 1992 see column 3, line 1 - line 25 ---	1-6
P,X	MICHAUD R. O.: "Efficient Asset Management, A Practical Guide to Stock Portfolio Optimization and Asset Allocation" June 1998 , HARVARD BUSINESS SCHOOL PRESS , BOSTON, MASSACHUSSETTS, USA, ISBN-0-87584-743-9 XP002105978 see page III - page XI see page 41, line 1 - page 81, line 31 ---	1-6

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[X] Further documents are listed in the continuation of box C.	[X] Patent family members are listed in annex.

° Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier document but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
15 June 1999	01/07/1999

Name and mailing address of the ISA
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	Authorized officer
	
	        Pedersen, N

C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category°	Citation of document, with indication,where appropriate, of the relevant passages	Relevant to claim No.
A	KING A J: "ASYMMETRIC RISK MEASURES AND TRACKING MODELS FOR PORTFOLIO OPTIMIZATION UNDER UNCERTAINTY" ANNALS OF OPERATIONS RESEARCH, vol. 45, no. 1/04, 1 December 1993, pages 165-177, XP000568230 see page 175, line 18 - page 176, line 23 ---	1-6
P,A	WO 98 13776 A (RCO SOFTWARE LIMITED ;MASCH VLADIMIR A (US)) 2 April 1998 see page 4, line 29 - page 6, line 24 see page 10, line 6 - line 35 see page 23, line 26 - page 24, line 24 see page 37, line 19 - page 38, line 29 ---	1-6
A	JENSEN D L ET AL: "FRONTIER: A GRAPHICAL INTERFACE FOR PORTFOLIO OPTIMIZATION IN A PIECEWISE LINEAR-QUADRATIC RISK FRAMEWORK" IBM SYSTEMS JOURNAL, vol. 31, no. 1, 1 January 1992, pages 62-70, XP000261866 see page 66, column 2, line 3 - page 67, column 2, line 3 ---	1-6
A	BELLITY L: "Fuzzy optimization applied to portfolio management" AI '94. FOURTEENTH INTERNATIONAL AVIGNON CONFERENCE. PROCEEDINGS, AI '94. FOURTEENTH INTERNATIONAL AVIGNON CONFERENCE, PARIS, FRANCE, 30 MAY-3 JUNE 1994, pages 469-476 vol.1, XP002105977 ISBN 2-910085-06-6, 1994, Nanterre, France, EC2, France see page 471, line 6 - line 42 see page 473, line 3 - line 38 see page 474, line 35 - page 475, line 7 -----	1-6

1

Form PCT/ISA/210 (continuation of second sheet) (July 1992)

INTERNATIONAL SEARCH REPORT

.ormation on patent family members

Patent document cited in search report		Publication date	Patent family member(s)		Publication date
US 5148365	A	15-09-1992	NONE		
WO 9813776	A	02-04-1998	AU	4483297 A	17-04-1998